Management of rain-fed reservoirs faces serious eco-hydrology problems in maintaining water availability and sustainable ecosystems in reservoirs. Water is a prerequisite for all life [Jorgensen, 2016], and plays an important role in supporting the achievement of the Sustainable Development Goals (SDGs) [Mugagga and Nabaasa, 2016]. Its availability is also very much needed during the COVID-19 pandemic, so its sustainable management is required to maintain food security [FAO, 2020; WHO, 2020]. The climate change has an impact on rainfall volatility [Kartono et al., 2020], thus affecting the dynamics of water volume growth in reservoirs. Therefore, it is important to analyze the dynamics of water volume growth to assess the potential for water abundance or scarcity [Chen, 2019].

The hydrological implications of the dynamics of water availability in reservoirs are often assessed according to the principles of input-output equilibrium [Araujo et al., 2006; Pandey et al., 2011; Fowe et al., 2015; Ali et al., 2017], or the concept of stock flow [Alifujiang et al., 2017]. Some of the mathematical models used to explain the dynamics of water volume growth in reservoirs are the water balance models [Bonacci and Roje, 2008; Pandey et al., 2011; Xi and Poh, 2013; Szporak-Wasilewska et al., 2015; Fowe et
The speed at which a reservoir reaches its water volume carrying capacity is important information in the sustainable reservoir management. The hydrological indicators that lead to the abundance or scarcity of water are the ecological data in dynamic modeling for early warning purposes [Burkhard et al., 2015; Forni et al., 2016] and for prediction [Mushar et al., 2019]. Mathematical modeling has the ability to predict based on historical data. Therefore, modeling of water volume growth dynamics is needed to analyze its ecological implications.

The growth model must consider the carrying capacity of the environment in accordance with the limited capacity of the reservoir. The model proposed in this study was constructed by modifying the generalized logistic model using Eq.(1) [Tsoularis, 2001].

$$\frac{dV_t}{dt} = rV_t^{a} \left[ 1 - \left( \frac{V_t}{K} \right)^c \right]^c$$

where $a$, $b$, $c$ is a positive real number, $V_t$ is a water volume (m$^3$) at time $t$ (days), $K$ is a water volume (m$^3$) carrying capacity, $r$/day is the intrinsic growth rate parameter.

Eq.(1) is a population growth model that depends on the carrying capacity of the environment [Jorgensen, 1994; Kribs-Zaleta, 2004; Chong et al., 2005; Peleg et al., 2007; Pinol and Banzon, 2011; Al-Saffar and Kim, 2017]. Some of the dynamic models that are modified from Eq.(1) are the Verhulst, Richards, and Gompertz models, which are distinguished based on the characteristics of the growth curve inflection points [Tsoularis, 2001].

The characteristic of the logistic growth curve is that a small population grows monotone, and at the point of infection, it then approaches asymptotically to a large constant value [Thornley et al., 2004; Idlango et al., 2017]. The growth function curve is S-shaped or sigmoid [Bradley, 2000; Miranda and Lima, 2010; Jin et al., 2018; Brilhante et al., 2019]. The maximum volume of water that can be reached by a reservoir without any disturbing influence on the availability of its resources is called the carrying capacity [Ross et al., 2005; Rogovchenko and Rogovchenko, 2009; Melica et al., 2014; Han et al., 2015; Jin et al., 2018; Brilhante et al., 2019]. The value of $r$ is a fundamental measure in the ecological and evolutionary phenomena that shows the rate at which the growth function curve reaches the carrying capacity of $K$ [Tsoularis, 2001; Miskinis and Vasiauskiene, 2017], shows the intrinsic ability of a population to grow [Shi et al., 2013; Cortes, 2016] or is the level of infection in epidemic phenomena [Bastita, 2020; Torrealba-Rodriguez et al., 2020].

The development of the application of the logistic growth model is used to explain the dynamics of biotic population growth. Some of the applications are a dynamic model of the growth of an organism or an increase in biomass with limited habitat resources [Chong et al., 2005; Peleg et al., 2007; Al-Saffar and Kim, 2017], a dynamic model of homogeneous population growth (single) in a biological system [Melica et al., 2014; Jin et al., 2018], vegetation dynamics models [Han et al., 2015], ecological models [Miskinis and Vasiauskiene, 2017], epidemic models and most recently for the COVID-19 pandemic [Bastita, 2020; Torrealba-Rodriguez et al., 2020] The intrinsic growth rate plays an important role in the analysis of the growth dynamics of the biotic population. Its role as an ecological parameter [Cortes, 2016] encourages the development of its application to the dynamics of a-biotic population growth. This article discusses the development of its application to the dynamics of water volume growth in a reservoir.

Therefore, the objective of this study was to analyze the dynamics of water volume growth in the reservoir based on its intrinsic growth rate. The study was conducted by modifying the generalized logistic growth model through the stages of verification, parameter estimation, data-based model validation to obtain a good model, and analyzing the intrinsic growth rate to assess the potential for local hydrometeorology disasters. This study was carried out in the Gembong Reservoir, Pati Regency, Indonesia in 2019–2020.

MATERIALS AND METHODS

Data collection

The mathematical model is a concept that describes the behavior of a real system quantitatively, which can be developed analytically and empirically. A good model fit was tested through the stages of verification, parameter estimation, and model

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*Ecological Engineering & Environmental Technology 2021, 22(4), 22–29*
validation [Jorgensen, 1994]. The data on daily water volume of the Gembong Reservoir for the period 2007-2018 is used as training, data in 2019 for testing, and data in 2020 for predicting. The proposed dynamic models, namely the Verhulst, Richards, Gompertz and Malthus modified models were tested for goodness based on the water volume data.

Model Development

The verification was carried out by making a curve of the water volume growth in each charging season to determine the shape of the curve and its geometric properties. The parameter estimation begins by solving the ordinary differential equations of each proposed model to obtain the growth function. The intrinsic growth rate parameter \( r \) is formulated and estimated according to the growth functions at each charging season. From the daily water volume data in each charging season, the initial water volume \( V_0 \) (\( m^3 \)), the water volume carrying capacity of the reservoir \( K \) (\( m^3 \)), and the duration of time (days) during the charging season, can be seen. Model validation was performed using the MAPE (Mean Absolute Percentage Error) criteria based on Eq. (2) to confirm the model goodness.

\[
MAPE = \frac{1}{n} \sum_{i=1}^{n} \left| \frac{Y_i - \hat{Y}_i}{Y_i} \right| \times 100\%
\]  

where \( Y_i \) is the \( i \)th water volume, \( \hat{Y}_i \) is a prediction of the \( i \)th water volume, and \( n \) is a sample size during the charging season, \( i = 1, 2, 3, \ldots n \). On the basis of the criteria by Lewis (1982), the forecast model with a smaller MAPE value is better [Hyndman and Koehler, 2006; Chen et al., 2008]. The analysis of the dynamics of water availability in reservoirs and the potential for local hydrometeorology disasters is based on the intrinsic growth rate characteristics of the selected model.

RESULTS AND DISCUSSION

Dynamic modeling of the water volume growth based on data

The field verification confirmed that charging in the Gembong Reservoir only occurs during the rainy season, the volume of water in the Gembong Reservoir is an accumulation of rainfall, and its infrastructure is in good condition [PJRRC, 2015]. The curve in Figure 1 is an example of the

![Fig. 1. Curve of the water volume in 2012](image)

Table 1. Models of growth and its intrinsic rate

<table>
<thead>
<tr>
<th>Model</th>
<th>Equation</th>
<th>Growth Function</th>
<th>( R )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Verhulst</td>
<td>( \frac{dV}{dt} = rV(1 - \frac{V}{K}) )</td>
<td>( V_i = \frac{K}{1 + e^{-r(t)}} )</td>
<td>( r = \frac{\ln(\frac{V_0}{K - V}) - (K - V_i)}{t} )</td>
</tr>
<tr>
<td>Richards ( b = 2 )</td>
<td>( \frac{dV}{dt} = rV(1 - \frac{V}{K}) )</td>
<td>( V_i = \frac{K}{1 - e^{-2r(1 - \frac{K}{V_0})^{1/2}}} )</td>
<td>( r = -\frac{1}{2t} \ln(\frac{V_0}{1 - \frac{K}{V_0}}) )</td>
</tr>
<tr>
<td>Gompertz</td>
<td>( \frac{dV}{dt} = rV(\ln(\frac{K}{V})) )</td>
<td>( V_i = K \exp[\ln(\frac{V_0}{K}) e^{-rt}] )</td>
<td>( r = -\frac{1}{t} \ln(\frac{V_0}{\ln(\frac{V_i}{K})}) )</td>
</tr>
<tr>
<td>Malthus modified</td>
<td>( \frac{dV}{dt} = r(K - V) )</td>
<td>( V_i = K + (V_0 - K)e^{-rt} )</td>
<td>( r = -\frac{1}{t} \ln(\frac{V - V_0}{V - K}) )</td>
</tr>
</tbody>
</table>
water volume growth curve in each charging season. Geometrically, there are three phases of the growth curve gradient, namely a small gradient at the beginning of the rainy season, then growing in the middle of the rainy season, and shrinking back asymptotically until the end of the rainy season [Thornley et al., 2004] or finally reaching a constant volume equivalent to volume. The water carrying capacity of reservoirs [Idlango et al., 2017]. The water volume growth curve resembles the logistic growth curve [Bradley, 2000; Tsoularis, 2001; Kribs-Zaleta, 2004; Chong et al., 2005; Miranda and Lima, 2010; Melica et al., 2014; Jin et al., 2018; Brilhante et al., 2019], so that the water volume growth curve in the Gembong Reservoir can be described by the logistic growth curve.

The growth function is a solution to the ordinary differential equation of the proposed model. Table 1 presents the formulations of the \( r \) parameter based on each of these solutions. The value of \( V_0 \) is known from each charging season, while \( K \) is the capacity of the reservoir. These \( V_0 \) and \( K \) values are substituted for the \( r \) equation for each of the proposed models, so that the \( r \) value for each \( t \) is obtained. Then, the mean value of \( r \) is calculated and used as the estimated value of \( r \) for each proposed model. Table 2 shows that the estimated value of \( r \) with the Verhulst model > Richards model > Gompertz model > modified Malthus model.

The estimated value of \( r \) in Table 2 is then substituted for each growth function in Table 1.

Table 2. Estimated parameter of \( r / \text{day} \) for each charging season

<table>
<thead>
<tr>
<th>Year</th>
<th>Verhulst</th>
<th>Richards ( b = 2 )</th>
<th>Gomperzt</th>
<th>Malthus modified</th>
</tr>
</thead>
<tbody>
<tr>
<td>2007</td>
<td>0.115</td>
<td>0.099</td>
<td>0.054</td>
<td>0.022</td>
</tr>
<tr>
<td>2008</td>
<td>0.069</td>
<td>0.056</td>
<td>0.034</td>
<td>0.016</td>
</tr>
<tr>
<td>2009</td>
<td>0.038</td>
<td>0.032</td>
<td>0.015</td>
<td>0.009</td>
</tr>
<tr>
<td>2010</td>
<td>0.054</td>
<td>0.028</td>
<td>0.052</td>
<td>0.051</td>
</tr>
<tr>
<td>2011</td>
<td>0.047</td>
<td>0.036</td>
<td>0.027</td>
<td>0.016</td>
</tr>
<tr>
<td>2012</td>
<td>0.057</td>
<td>0.047</td>
<td>0.030</td>
<td>0.015</td>
</tr>
<tr>
<td>2013</td>
<td>0.099</td>
<td>0.085</td>
<td>0.052</td>
<td>0.026</td>
</tr>
<tr>
<td>2014</td>
<td>0.068</td>
<td>0.053</td>
<td>0.039</td>
<td>0.022</td>
</tr>
<tr>
<td>2015</td>
<td>0.028</td>
<td>0.029</td>
<td>0.016</td>
<td>0.007</td>
</tr>
<tr>
<td>2016</td>
<td>0.082</td>
<td>0.063</td>
<td>0.046</td>
<td>0.026</td>
</tr>
<tr>
<td>2017</td>
<td>0.084</td>
<td>0.068</td>
<td>0.044</td>
<td>0.022</td>
</tr>
<tr>
<td>2018</td>
<td>0.030</td>
<td>0.025</td>
<td>0.016</td>
<td>0.008</td>
</tr>
<tr>
<td>2019</td>
<td>0.101</td>
<td>0.082</td>
<td>0.054</td>
<td>0.028</td>
</tr>
<tr>
<td>Average</td>
<td>0.067</td>
<td>0.055</td>
<td>0.037</td>
<td>0.021</td>
</tr>
</tbody>
</table>

Table 3. MAPE for each charging season (%)

<table>
<thead>
<tr>
<th>Year</th>
<th>Verhulst</th>
<th>Richards ( b = 2 )</th>
<th>Gomperzt</th>
<th>Malthus modified</th>
</tr>
</thead>
<tbody>
<tr>
<td>2007</td>
<td>14</td>
<td>18</td>
<td>12</td>
<td>12</td>
</tr>
<tr>
<td>2008</td>
<td>15</td>
<td>11</td>
<td>12</td>
<td>12</td>
</tr>
<tr>
<td>2009</td>
<td>65</td>
<td>40</td>
<td>38</td>
<td>38</td>
</tr>
<tr>
<td>2010</td>
<td>0.3</td>
<td>222</td>
<td>43</td>
<td>43</td>
</tr>
<tr>
<td>2011</td>
<td>10</td>
<td>30</td>
<td>30</td>
<td>30</td>
</tr>
<tr>
<td>2012</td>
<td>5</td>
<td>20</td>
<td>17</td>
<td>17</td>
</tr>
<tr>
<td>2013</td>
<td>5</td>
<td>14</td>
<td>18</td>
<td>18</td>
</tr>
<tr>
<td>2014</td>
<td>8</td>
<td>11</td>
<td>13</td>
<td>13</td>
</tr>
<tr>
<td>2015</td>
<td>32</td>
<td>55</td>
<td>19</td>
<td>19</td>
</tr>
<tr>
<td>2016</td>
<td>3</td>
<td>11</td>
<td>13</td>
<td>13</td>
</tr>
<tr>
<td>2017</td>
<td>10</td>
<td>11</td>
<td>14</td>
<td>14</td>
</tr>
<tr>
<td>2018</td>
<td>25</td>
<td>53</td>
<td>54</td>
<td>54</td>
</tr>
<tr>
<td>Average</td>
<td>16</td>
<td>41</td>
<td>17</td>
<td>17</td>
</tr>
</tbody>
</table>
Validation of each model was carried out by calculating the MAPE training value ($tr$) and the MAPE testing value ($ts$) to determine the accuracy of the model. Table 3 presents the $tr$ and $ts$ values of each of the proposed models.

Table 3 shows that the Richards model with $b = 2$ has consistently smaller $tr$ and $ts$ values than the other three models. Thus, the Richards model with $b = 2$ is the best model of the other three models. The 2016 charging season has little MAPE consistency, so that the estimated parameter value $r = 0.063$/day in Table 2 based on the Richards model with $b = 2$ is chosen as the parameter value $r$. Therefore, the Richards growth model with $b = 2$ and $r = 0.063$/day is a good model [Hyndman and Koehler, 2006; Chen et al., 2008] to explain the dynamics of water volume growth in the Gembong Reservoir. Figure 2 presents the growth curve of the Richards model with $b = 2$ and $r = 0.063$/day and the growth curve for daily water volume for the 2020 charging season.

The dynamic analysis of water volume growth

Figure 2 shows that the curve of the Richards model with $b = 2$ is good enough to illustrate the growth curve of water volume in the Gembong Reservoir in the 2020 charging season. The ecological implication of the phenomenon of water volume growth in the reservoir in each rainy season has three growth phases, as shown by the curve in Figure 2. Rainfall at the beginning of the rainy season is relatively small, so the intrinsic growth rate of reservoir water volume is also small. Rainfall becomes larger in the mid-rainy season phase. The loss of rainwater caused by the interception is getting smaller, so that the rain falling on the land has the potential to flow into the reservoir storage. The intrinsic growth rate of water volume in this phase is greater, which then reaches the inflection point [Tsoularis, 2001], and finally shrinks asymptotically to near zero. The $r$ parameter in Table 2 fluctuates from year to year without following a certain trend pattern. The characteristics of the $r$ parameter are in line with the characteristics of local rainfall [Kartono et al., 2020]. The Richards growth function in Table 1 fulfills the asymptotic nature, using Eq. 3.

$$\lim_{t \to \infty} V(t) = K$$  \hspace{1cm} (3)

Geometrically from Eq.(3), the line $V = K$ is an asymptotic line of the $V_t$ curve, meaning that the growth in reservoir water volume reaches its carrying capacity under saturated conditions [Kribs-Zelita, 2004]. The higher the $r$ value, the greater the speed of the reservoir to reach saturation. This shows that the dynamic model of biotic population growth [Jorgensen, 1994; Peleg, 2006; Al-Saffar and Kim, 2017; Jin et al., 2018; Batista, 2020; Torrealba-Rodriguez et al., 2020] can be developed as a dynamics model of a-biotic population growth (water volume). The reproduction rate in a biotic population is defined by the rate of increase in the a-biotic population. The logistic growth models can reveal the dynamic properties of water volume growth in reservoirs, although the goodness of the model for prediction depends on the goodness or quality of empirical data [Batista, 2020].

Ecological implications of the dynamics of water volume growth based on the intrinsic growth rate

The average intrinsic growth rate of the Richards model of 0.055/day can be used as an early warning indicator [Burkhard et al., 2015; Forni et al., 2016], which indicates the potential for water abundance in the reservoir [Mushar et al., 2019]. The potential for flood disasters
triggered by the abundance of water is becoming higher along with the shrinking capacity of the reservoir due to increased sedimentation.

There are two phases of risk to be aware of, namely the growth phase and the saturation phase during the reservoir water charging process. If the value of $r$ is greater in the growth phase, then $t$ is smaller. The reservoir reaches its water volume carrying capacity faster (the reservoir is full), and the reservoir experiences a saturation condition for a longtime. In this phase, some of the reservoir water will overflow through the spillway when it rains. The longer the remaining rainy season, the greater the water overflow through the spillway. This condition increases the environmental pressure on the resilience of the reservoir building which can have fatal consequences [Harsoyo, 2010], so that normalization of the reservoir needs to be done as a mitigation strategy against potential flooding.

Conversely, if the value of $r$ is becoming smaller, then $t$ is becoming greater, meaning that the reservoir takes longer to reach the water volume carrying capacity. Under this condition, the reservoir does not take too long to fully store water, so that the overflow of water through the spillway is smaller or does not occur. The potential for scarcity of water can trigger a meteorologist drought when the value of $r$ is very small. Such a trend cannot be understood as having the potential for flooding, given that 75% of local rainfall occurs with high concentrations [Kartono et al., 2020].

CONCLUSIONS

The Richards model with $b = 2$ and $r = 0.063$/day is a good model to explain the dynamics of water volume growth in the Gembong Reservoir. The intrinsic growth rate $r = 0.063 /$ day is greater than the average growth rate. The greater the value of the ecological parameter $r$, the faster the growth function curve reaches its saturation value. Thus, the ecological implication of these dynamics of water volume growth is that reservoirs experience an abundance of water during the charging season. The awareness of the potential for flooding needs to be increased. Reservoir normalization can be prioritized as a mitigation strategy for potential flood disasters. It is urgent to reduce the volume of sediment in the Gembong Reservoir.

The results of this study indicate a new contribution from the application of the logistic growth model in the real world phenomena and enriching scientific knowledge, namely the application of mathematical models in eco-hydrology studies. The intrinsic growth rate applies not only to the growth dynamics of biotic populations, but also to the dynamics of a-biotic population growth. The methodology for obtaining a good model through the stages of verification, parameter estimation, and model validation based on empirical data can be applied to similar reservoirs. The development of the application of a logistic model to another a-biotic population growth phenomenon opens up the opportunities for further research.

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