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Discharge Coefficient of Rectangular Broad-Crested Weirs in Narrow Channels with High Relative Length of the Threshold

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ABSTRACT

Broad-crested weirs (BCW) are commonly used elements of stormwater systems and different open-channel hydraulic structures. Specific features of stormwater drainage channels are small width, low flow depths and, accordingly, small overflow heads at weirs. Dependences of the discharge coefficient of narrow (b = 0.224 m) rectangular sharp-edged broad-crested weirs with vertical walls, threshold height of 0.05 m and threshold length of 0.05–0.2 m were obtained experimentally. The experimental values of the discharge coefficient were approximated by the power-law functions of relative length of the weir. At large values of the relative length of the threshold (L/h> 10), for all weirs was obtained the same tendency of decreasing the discharge coefficient with increasing L/h ratio that can be explained by the enlargement of the hydraulic friction along the weir with increasing L/h ratio.

Keywords: broad-crested weir, discharge coefficient, narrow channel, relative length of the threshold

INTRODUCTION

The quality of hydraulic modelling of stormwater systems on urban areas depends on an adequate and representative collection of three main groups of input data. The first group consists of the climatic data about rainfall parameters in the form of dependences of rainfall intensity on its duration and frequency [Banaszkiewicz et al., 2009; Zhuk et al., 2021]. The second group contains the data about the distribution of coverings at the runoff catchment, about soil types, as well as the features of the relationships between subcatchments and stormwater network, which can be represented as a functional dependence between total and effective imperviousness [Lee & Heaney, 2003; Zhuk et al., 2020b]. The third group of input data includes the geospatial data on the location of individual elements of the stormwater system, as well as their main hydraulic parameters [Aqnouy et al., 2021].

The accuracy of the estimation of hydraulic resistance of individual elements of the stormwater systems has a significant impact on the quality of the corresponding computer hydraulic model as a whole. Broad-crested weirs (BCW) are commonly used elements of stormwater systems, including structures for the detention, regulation and retention of runoff from urban catchments. Initial and ending sections of drainage channels, stormwater inlets, various input and output non-pressure devices of regulating structures, work, from the hydraulic point of view, as BCW [Cosco et al., 2020].

A specific feature of stormwater drainage channels is their small width, which varies, as usual, in the range from 100 mm to 400 mm; approximately the same values of the width of the surface flow are at the stormwater inlets. Another peculiarity of surface runoff is low flow depths and, accordingly, small overflow heads at weirs.

The hydraulic calculation of BCW is sufficiently fully and deeply substantiated and verified, the results of numerous studies are summarised in regulatory documents. According to [ASTM D5614–94, 2008; ISO 3846:2008, 2016;

Zachoval et al., 2014] the volumetric flow rate through BCW is expressed as follows:

$$Q = \left(\frac{2}{3}\right)^{\frac{3}{2}} Cb\sqrt{g}h^{3/2} \quad (1)$$

where: C – discharge coefficient using the upstream overflow head h;
b – width of the weir between the vertical side-walls.

Other researchers [Hager & Schwalt, 1994; Zhuk et al., 2020a] determine the flow rate using the formula of another structure:

$$Q = C_d b\sqrt{2gH^3} \quad (2)$$

where C_d – discharge coefficient using the total head H;
$H = h + \frac{\alpha V_0^2}{2g}$ – total head, m;
α – coefficient of kinetic energy (Coriolis coefficient);
V_0 – average velocity in upstream cross-section.

In the most general formulation discharge coefficient of BCW depends on several geometric simplexes, as well as the Reynolds, Froud and Weber numbers:

$$C = f(L/h;\ P/h;\ b/h;\ k/h;\ \text{Re};\ \text{Fr};\ \text{We}) \quad (3)$$

where: L – length of weir in the flow-wise direction;
P – height of the weir;
k – roughness of the weir walls;
Re, Fr, and We – respectively Reynolds, Froude and Weber numbers.

In regulatory documents [ASTM D5614–94, 2008; ISO 3846:2008, 2016] values of the discharge coefficient of BCW are considered only as a function of the relative height and relative length of the threshold (*P/h* and *L/h*, respectively). The geometrical parameters of the BCW according to [ASTM D5614–94, 2008] should be in the following limits: 1) $h \geq 0.06$ m, $b \geq 0.3$ m and $P \geq 0.15$ m – to avoid the effects of viscosity and surface tension; 2) $0.1 < L / P < 4$, $0.6 < L / h < 10$, so as beyond these ranges there are no data on practical calibration; 3) $P / h > 0.6$ to avoid unstable water levels.

According to [Hager & Schwalt, 1994], the discharge coefficient depends exclusively on the relative weir length *L/H*, thus, effects of viscosity and surface tension may be neglected:

$$C_d = \frac{9}{7} C_{d.0}\left[1 - \frac{2}{9\left(1 + (H/L)^4\right)}\right] \quad (4)$$

where: $C_{d0} = 0.326$ when $H/L \leq 2.2$, and
$C_{d0} = 0.329$ if $H/L > 2.2$.

At the same time, hydraulic studies of BCW were continued in other specific working parameters, beyond the ranges recommended by [ASTM D5614–94, 2008; ISO 3846:2008, 2016]. The geometric parameters of previous experimental studies of narrow BCW with a threshold width $b \leq 0.3$ m are presented in the Table 1.

By processing the experimental results a number of empirical dependences were obtained to determine the discharge coefficient of narrow BCW.

For a rectangular BCW assuming critical flow on the weir crest with no boundary layer

Table 1. Geometric parameters of experimental studies of narrow BCW with a threshold width $b \leq 0.3$ m

Investigator	Weir width *b*, m	Overflow head *h*, m	Weir thickness *L*, m	Weir height *P*, m	Ratio *L/P*	Ratio *L/h*
[Sreetharan, 1983]	0.27–0.51	0.037–0.290	0.08–0.90	0.08–0.20	1–4.5	2.16–3.1
[Tim, 1986]	0.25	0.025–0.122	0.31	0.1	3.1	2.53–12.3
[Gonzalez and Chanson, 2007]	0.25–1	0.015–0.29	0.42–0.88	0.06–0.99	0.9–7	3.03–28
[Goodarzi et al., 2012]	0.25	0.115–0.195	0.6	0.25	2.4	3.07–5.26
[Bijankhan et al., 2013]	0.002–0.2	0.0124–0.0717	0.6	0.1–0.2	0.02–2.82	0.06–50
[Hovany, 2019]	0.1	0.0034–0.0646	0.095–0.5	0.0678–0.0711	1.4–7.03	2.9–50

development [Tim, 1986] obtained next expression for the discharge coefficient:

$$C=\left(\frac{27}{16}\left(\frac{\left(\frac{h+P}{h}\right)^2\cdot\left(\frac{d_3}{h}\right)^2\cdot K_p\left(\frac{2h+P}{h}\right)\left(\frac{P}{h}\right)}{\left(\frac{h}{h+P}\right)\cdot\left(\frac{h}{d_3}\right)}\right)\right)^{\frac{1}{2}} \quad (5)$$

where: K_p – coefficient of deviation of pressure from hydrostatic distribution;
d_3 – depth of flow at downstream control section of rectangular BCW.

The results of an experimental study of BCW with a rounded crest in a wide range of widths b = 0.25–1 m and height P = 0.06–0.99 m are presented in the paper [Gonzalez and Chanson, 2007]. The effect of viscosity and surface tension similarly to [Hager & Schwalt, 1994] was neglected. It was found that the discharge coefficient of narrow BCW in the range $2.63 \leq L/h \leq 8.33$ can be approximated by a linear dependence

$$C=1.013-0.228\cdot\frac{h}{L} \quad (6)$$

Impact of upstream face slope angles on discharge coefficient of rectangular BCW was investigated in detail by [Goodarzi et al., 2012] and new correction factor C_r is proposed to take into account the influence of angle.

In [Bijankhan et al., 2014] the following discharge coefficients for the rectangular sharp-edged BCW depending the relative length of the weir were obtained:
for the range $0.5 < L/h < 2.5$:

$$C=0.9309+\left(\frac{L}{h}\right)^{-0.1839} \quad (7)$$

for $2.5 < L/h < 10$:

$$C=4.2003\left(\frac{h}{h+P}\right)^2-2.5966\left(\frac{h}{h+P}\right)+1.3563 \quad (8)$$

for $L/h > 10$:

$$C=0.532\left(\frac{h}{h+P}\right)^{-0.342} \quad (9)$$

The object of the study in [Hovany, 2019] were rectangular BCW with a width of b = 100 mm and different lengths in the range of low overflow heads $h \leq 64.6$ mm. It was found that viscosity and water surface tension have impact on the discharge coefficient of the BCW only if $L/h \geq 10$.

The purpose of this study was to obtain experimental dependences of the discharge coefficient of narrow ($b \leq 0.3$ m) rectangular sharp-edged BCW with vertical walls at low upstream overflow heads that corresponds to especially high relative length of the threshold.

MATERIALS AND METHODS

The experiments were performed in the Research Hydraulic Laboratory of the Department of Hydraulic and Water Engineering in the Lviv Polytechnic National University. Investigated BCW with a height of threshold P = 50 mm and length L = 50–200 mm were installed in a hydraulic channel with width b = 224 mm. A series of experimental studies to find discharge coefficient of BCW in the range L/P = (1–4) and variable head h in the range between 5.0 mm to 58.5 mm; minimum overflow head corresponds to the results obtained by [Hovany, 2019].

Flow rate Q was found using a standard V-notch measuring weir that was calibrated by using the volumetric method, with a standard error σ = 0.018 compared to the results of King's formula:

$$Q = 1.343\, h_m^{2.47} \quad (10)$$

where: h_m is the head on the measuring weir, m.

The experimental values of the discharge coefficient of BCW were calculated using the equation (1). The maximum relative error of the experimental determination of the BCW discharge coefficient was 1.0–3.2% depending on the value of the overflow head on the weir, that consistent with the specified permissible errors in the standard [ASTM D5640–95, 2014].

RESULTS AND DISCUSSION

The results of the experimental studies obtained for BCW with a different length of weir L = 0.05–0.2 m are presented below. The height of all of the weirs P = 0.05 m. Investigated ranges of the main input parameters for BCW are presented in the Table 2.

The experimental results are presented as graphical dependencies of the discharge coefficient C on the relativ length of the weir L/h (Fig. 1).

The obtained results were compared with the values of the discharge coefficient C according to the widely used formula (4) (Hager & Schwalt, 1994). The coefficient C_d in equation (2) was converted into the discharge coefficient C in the structure of formula (1).

The obtained experimental results are well described by power-law functions. For the length of the weir L = 0.05 m (L/P = 1) it is obtained:

$$C = 1.012(L/h)^{-0.100} \tag{11}$$

Table 2. Parameters of investigated BCW (b = 0.224 m; P = 0.05 m)

Length of weir L, m	Ratio L/P	Overflow head $h \cdot 10^{-2}$, m	Ratio L/h	Flow rate Q, dm³/s	Reynolds number Re	Froude number Fr
0.05	1	0.71–4.99	0.66–7.04	0.23–0.70	664–10675	0.0044–0.0833
0.1	2	0.89–5.32	1.88–11.24	0.29–4.43	839–10264	0.0054–0.0696
0.15	3	0.50–5.85	2.56–26.79	0.09–5.03	277–11354	0.0011–0.0734
0.2	4	0.51–3.83	5.22–39.22	0.04–2.49	132–6208	0.0003–0.0421

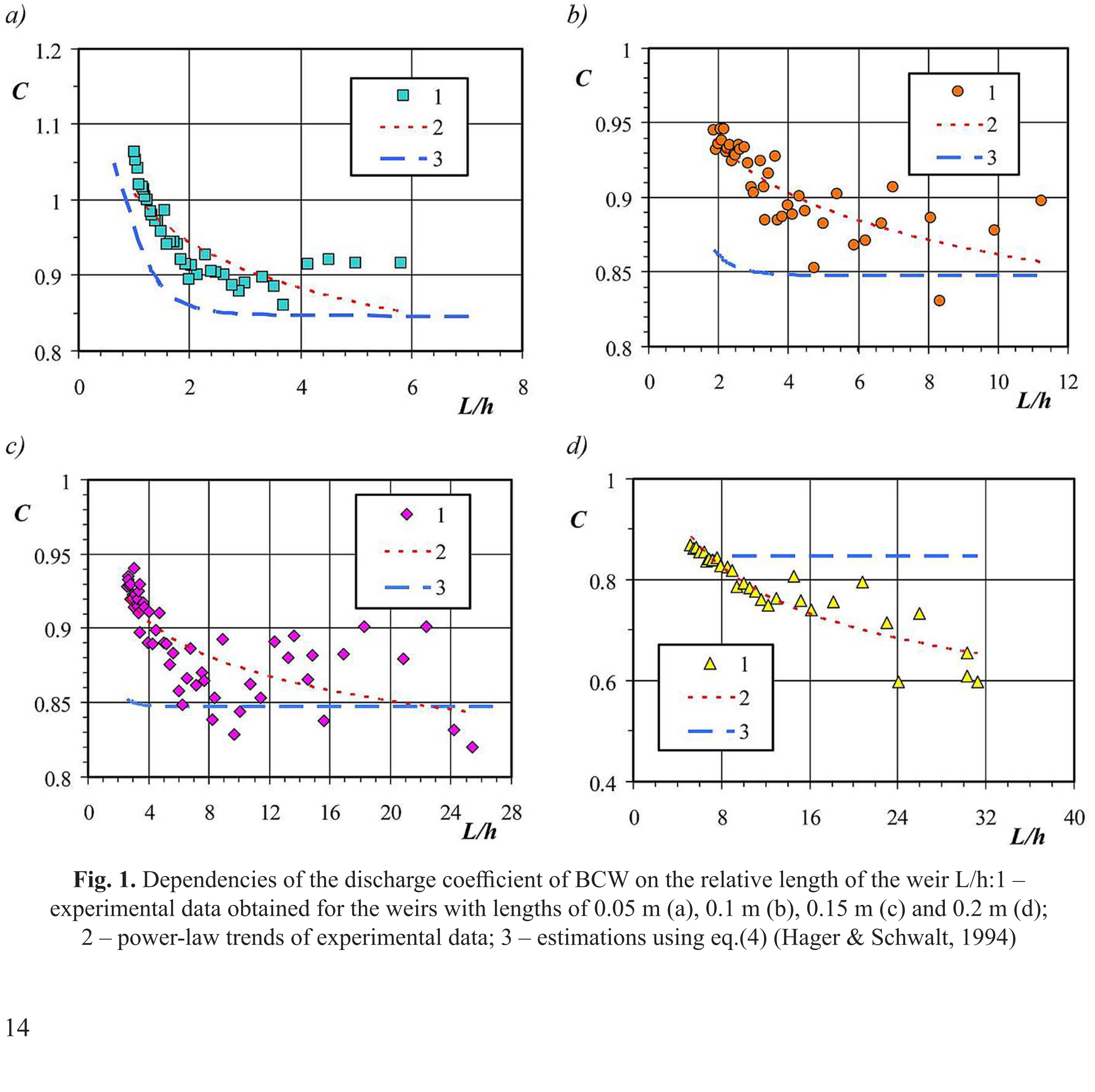

Fig. 1. Dependencies of the discharge coefficient of BCW on the relative length of the weir L/h:1 – experimental data obtained for the weirs with lengths of 0.05 m (a), 0.1 m (b), 0.15 m (c) and 0.2 m (d); 2 – power-law trends of experimental data; 3 – estimations using eq.(4) (Hager & Schwalt, 1994)

for $L = 0.1$ m $(L/P = 2)$:

$$C = 0.969(L/h)^{-0.051} \quad (12)$$

for $L = 0.1$ m $(L/P = 2)$:

$$C = 0.954(L/h)^{-0.038} \quad (13)$$

for $L = 0.1$ m $(L/P = 2)$:

$$C = 1.153(L/h)^{-0.162} \quad (14)$$

CONCLUSIONS

Dependencies of the discharge coefficient of narrow (b = 0.224 m) rectangular sharp-edged broad-crested weirs with vertical walls, threshold height of 0.05 m and threshold length of 0.05–0.2 m were obtained experimentally. Special attention was paid to the range of low upstream overflow heads (h = 5–58.5 mm) that corresponds to the relative length of weirs L/h = 0.66–39.22.

The experimental values of the discharge coefficient are approximated by the power-law functions (11)–(14). At large values of the relative length of the threshold (L/h > 10) for all weirs obtained the same tendency of decreasing the discharge coefficient with increasing L/h ratio that can be explained by the enlargement of the hydraulic friction along the weir with increasing L/h ratio.

The comparison with the results, obtained by the widely used method presented in (Hager & Schwalt, 1994), shows that discharge coefficient of rectangular BCW, placed in narrow channels, at small overflow heads depends on both the relative length of the weir L/h and its relative height P/h, which confirms and complements the recommendations of standards [ASTM D5614–94, 2008; ISO 3846:2008, 2016].

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